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(54) SKILL TRANSFER FROM A PERSON TO A ROBOT

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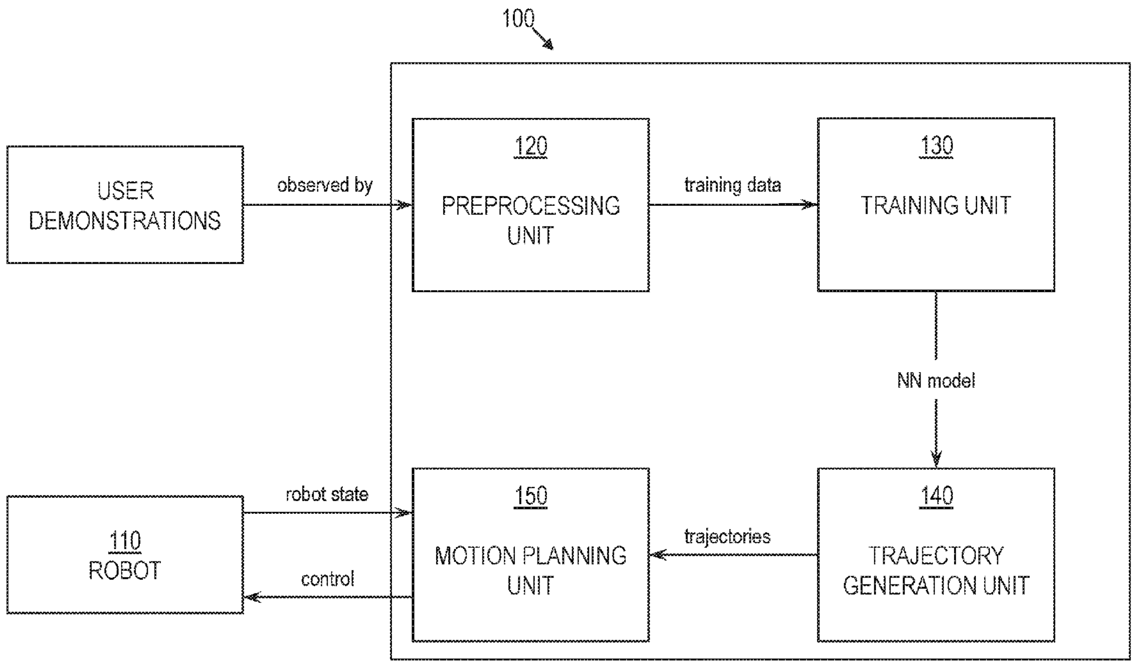
(57) **ABSTRACT**

A computer-implemented method includes recording one or more demonstrations of a task performed by a user. Movements of one or more joints of the user are determined from the one or more demonstrations. By a computer processor, a neural network or Gaussian mixture model incorporating one or more contraction analysis constraints is learned, based on the movements of the one or more joints of the user, the one or more contraction analysis constraints representing motion characteristics of the task. A first initial position of a robot is determined. A first trajectory of the robot is determined to perform the task, based at least in part on the neural network or Gaussian mixture model and the first initial position.



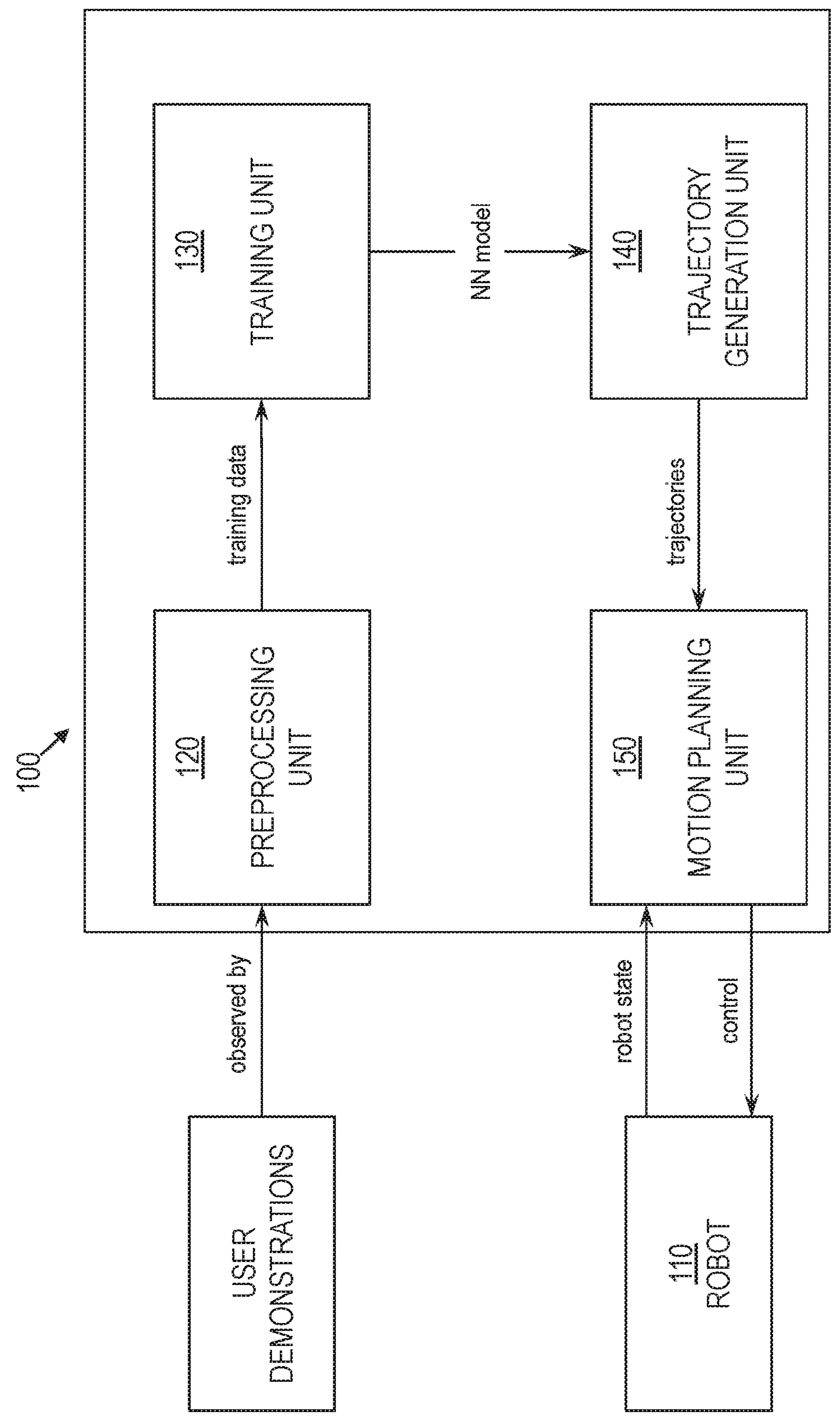

100
120 PREPROCESSING UNIT
training data
130 TRAINING UNIT
NN model
140 TRAJECTORY GENERATION UNIT
trajectories
150 MOTION PLANNING UNIT
USER DEMONSTRATIONS
observed by
110 ROBOT
robot state
control

Fig. 1

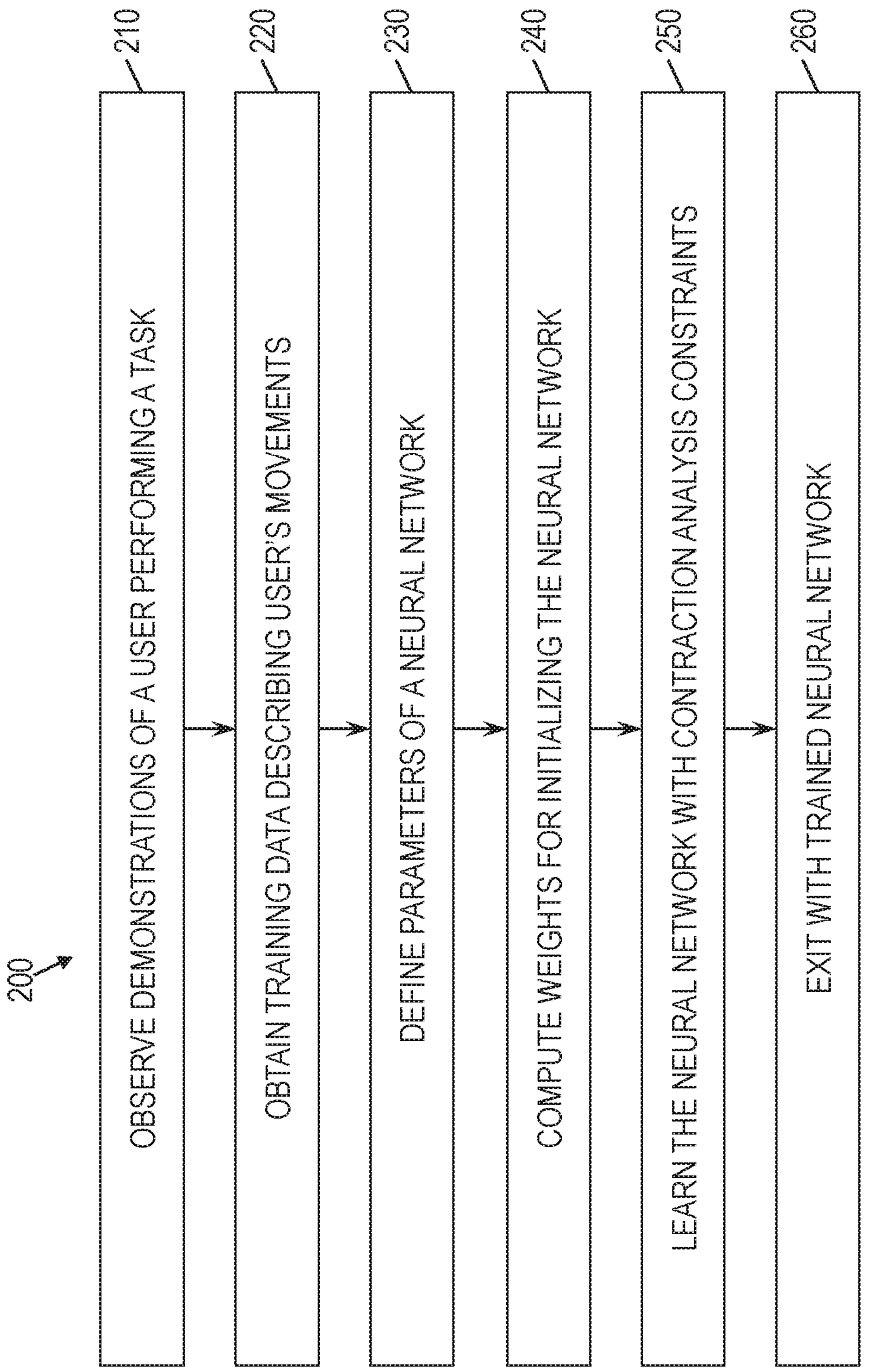

200
210
OBSERVE DEMONSTRATIONS OF A USER PERFORMING A TASK
220
OBTAIN TRAINING DATA DESCRIBING USER'S MOVEMENTS
230
DEFINE PARAMETERS OF A NEURAL NETWORK
240
COMPUTE WEIGHTS FOR INITIALIZING THE NEURAL NETWORK
250
LEARN THE NEURAL NETWORK WITH CONTRACTION ANALYSIS CONSTRAINTS
260
EXIT WITH TRAINED NEURAL NETWORK

Fig. 2

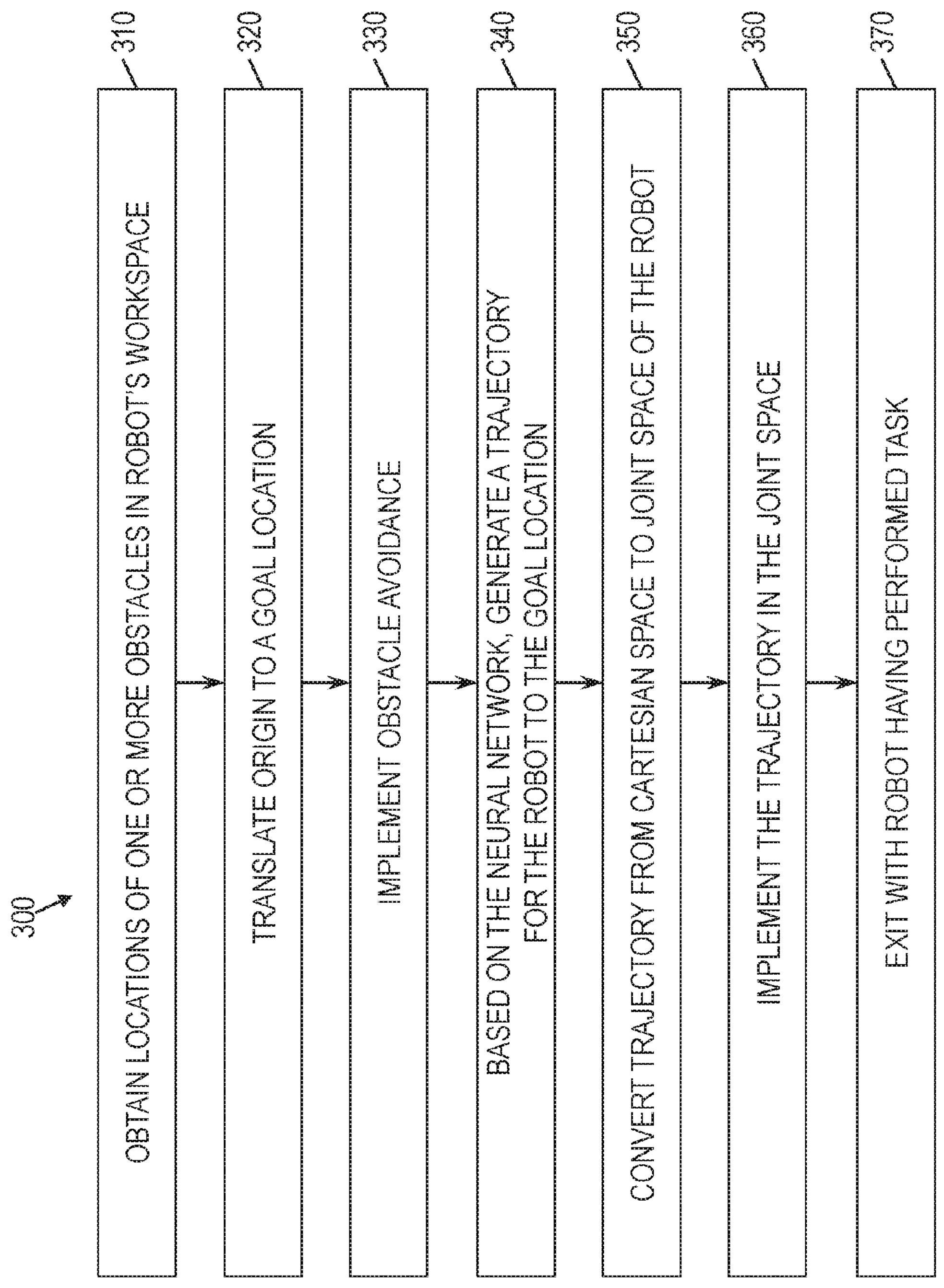

300
310
OBTAIN LOCATIONS OF ONE OR MORE OBSTACLES IN ROBOT'S WORKSPACE
320
TRANSLATE ORIGIN TO A GOAL LOCATION
330
IMPLEMENT OBSTACLE AVOIDANCE
340
BASED ON THE NEURAL NETWORK, GENERATE A TRAJECTORY FOR THE ROBOT TO THE GOAL LOCATION
350
CONVERT TRAJECTORY FROM CARTESIAN SPACE TO JOINT SPACE OF THE ROBOT
360
IMPLEMENT THE TRAJECTORY IN THE JOINT SPACE
370
EXIT WITH ROBOT HAVING PERFORMED TASK

Fig. 3

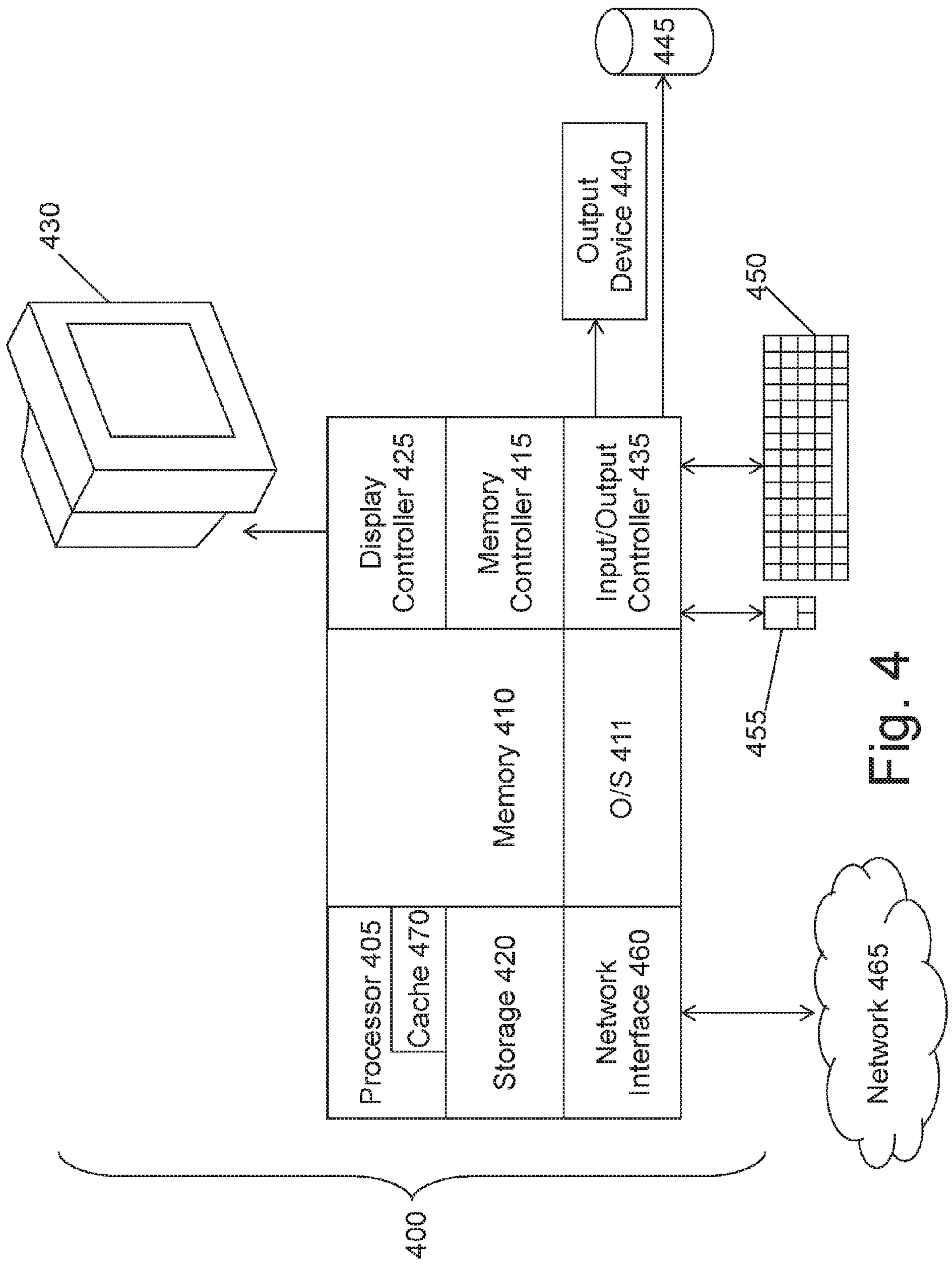

430
445
Output Device 440
450
Display Controller 425
Memory Controller 415
Input/Output Controller 435
Memory 410
O/S 411
455
Processor 405
Cache 470
Storage 420
Network Interface 460
Network 465
400

Fig. 4

SKILL TRANSFER FROM A PERSON TO A ROBOT

CROSS-REFERENCE TO RELATED APPLICATIONS

[0001] The present application claims priority to U.S. Provisional Application No. 62/366,659, filed on Jul. 26, 2016, the contents of which are incorporated by reference herein in their entirety.

BACKGROUND

[0002] Embodiments relate to robotics and, more specifically, to skill transfer from a person to a robot.

[0003] Learning from demonstration is an important problem in the context of training robots using non-expert operators, i.e., operators who are not equipped to reprogram the robots. For instance, to teach a task to a robot, the robot can be given the ability to learn motion from demonstrations of the task performed by a user. Thus, the user, despite being a non-expert operator, can teach the task to the robot by demonstrating the task. This technique may enable robots to perform, for example, in manufacturing contexts or as assistants to the elderly.

SUMMARY

[0004] According to an embodiment of this disclosure, a computer-implemented method includes recording one or more demonstrations of a task performed by a user. Movements of one or more joints of the user are determined from the one or more demonstrations. A neural network or Gaussian mixture model incorporating one or more contraction analysis constraints is learned by a computer processor based on the movements of the one or more joints of the user. The one or more contraction analysis constraints represent motion characteristics of the task. A first initial position of a robot is determined. A first trajectory of the robot to perform the task is determined based, at least in part, on the neural network or Gaussian mixture model and the first initial position.

[0005] In another embodiment, a system includes a memory having computer readable instructions and one or more processors for executing the computer readable instructions. The computer readable instructions include recording one or more demonstrations of a task performed by a user. Further according to the computer readable instructions, movements of one or more joints of the user are determined from the one or more demonstrations. A neural network or Gaussian mixture model incorporating one or more contraction analysis constraints is learned, based on the movements of the one or more joints of the user. The one or more contraction analysis constraints represent motion characteristics of the task. A first initial position of a robot is determined. A first trajectory of the robot to perform the task is determined based, at least in part, on the neural network or Gaussian mixture model and the first initial position.

[0006] In yet another embodiment, a computer program product for transferring a skill to a robot includes a computer readable storage medium having program instructions embodied therewith. The program instructions are executable by a processor to cause the processor to perform a method. The method includes recording one or more demonstrations of a task performed by a user. Further according to the method, movements of one or more joints of the user are determined from the one or more demonstrations. A neural network or Gaussian mixture model incorporating one or more contraction analysis constraints is learned, based on the movements of the one or more joints of the user. The one or more contraction analysis constraints represent motion characteristics of the task. A first initial position of a robot is determined. A first trajectory of the robot to perform the task is determined based, at least in part, on the neural network or Gaussian mixture model and the first initial position.

[0007] Additional features and advantages are realized through the techniques of the present invention. Other embodiments and aspects of the invention are described in detail herein and are considered a part of the claimed invention. For a better understanding of the invention with the advantages and the features, refer to the description and to the drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

[0008] The subject matter regarded as the invention is particularly pointed out and distinctly claimed in the claims at the conclusion of the specification. The foregoing and other features and advantages of the invention are apparent from the following detailed description taken in conjunction with the accompanying drawings in which:

[0009] FIG. **1** is a block diagram of a learning system used to control a robot, by learning a skill through one or more user demonstrations of a task, according to one or more embodiments of this disclosure;

[0010] FIG. **2** is a flow diagram of a method for learning semi-contracting or partially contracting nonlinear dynamics from the one or more demonstrations for the purpose of robot motion planning, according to one or more embodiments of this disclosure;

[0011] FIG. **3** is a flow diagram of a method for causing a robot to perform the task, according to one or more embodiments of this disclosure; and

[0012] FIG. **4** is a block diagram of a computer system for implementing some or all aspects of the learning system, according to one or more embodiments of this disclosure.

DETAILED DESCRIPTION

[0013] Some embodiments of this disclosure are learning systems that are based on a method of contracting dynamic system primitive (CDSP). Using CDSP, the learning system may learn motion dynamics, specifically a complex human motion dynamic model, using a neural network (NN) or a Gaussian mixture model (GMM) subject to motion trajectory constraints of a task, such as a reaching task. According to some embodiments, a human arm's reaching motion is modeled using a dynamic system (DS) $\dot{x}=f(x)$, where $f(x)$ is represented using a NN or GMM that is learned based on one or more demonstrations of the task by a human user.

[0014] In some embodiments, the problem of learning motion dynamics is formulated as a parameter learning problem of a NN or GMM under stability constraints given by contraction analysis of nonlinear systems. The contraction analysis may yield global exponential stability, in the form of a globally semi-contracting or partially contracting function, of the nonlinear systems.

[0015] There may be various benefits to learning a globally semi-contracting or partially contracting function in this context. For instance, motion trajectories may converge to a goal location from various initial conditions. Thus, regardless of the initial conditions when a robot performs a task, the robot may reach the desired goal location. A further benefit may be that, with the addition of an obstacle avoidance feature, trajectories generated for the robot may still converge to the goal location in the existence of obstacles.

[0016] FIG. **1** is a block diagram of a learning system **100** used to control a robot **110**, by learning a skill through one or more user demonstrations, according to one or more embodiments of this disclosure. As shown, the learning system **100** may include a preprocessing unit **120**, a training unit **130**, a trajectory generation unit **140**, and a motion planning unit **150**. Generally, the preprocessing unit **120** may observe one or more demonstrations of a task performed by a user and may perform preprocessing tasks, such as obtaining position, velocity, and acceleration of the human's movements in performing the task; the training unit **130** may define parameters of the NN or GMM and may obtain weights for the NN or GMM; the trajectory generation unit **140** may observe a state of the robot and generate a trajectory enabling the robot to perform the task, based on the NN or GMM; and the motion planning unit **150** may apply the trajectory to the robot **110**, thus causing the robot **110** to perform the task. The preprocessing unit **120**, training unit **130**, trajectory generation unit **140**, and motion planning unit **150** may include hardware, software, or a combination of both. Further, although these units are illustrated as separate in FIG. **1**, one of skill in the art will understand that they may share hardware, software, or both.

[0017] In some embodiments, as will be described further below, the learning system **100** may formulate an optimization problem, which may be used to compute weights of the NN or GMM subject to one or more contraction conditions of underlying dynamics. One or more contraction constraints may yield a state-dependent matrix inequality condition (i.e., a contraction inequality constraint), which may be nonconvex in the weights of the NN or GMM. In the case of a NN, the contraction inequality constraint may be reformulated as linear inequality conditions (i.e., linear inequality constraints) by assuming that a contraction metric is a constant and the number of neurons in a hidden layer of the NN is equal to the number of inputs. In the case of a GMM, the contraction inequality constraint may be reformulated as a polynomial matrix inequality constraint by assuming that the elements of a contraction metric are polynomial functions in the state.

[0018] Further, in some embodiments, the learning system **100** may use sequential quadratic programming (SQP), in a novel learning algorithm, subject to the relaxed contraction constraints. The learning system **100** may select good initial conditions for the constrained SQP based on the solutions obtained by solving an unconstrained optimization problem first.

[0019] In some embodiments, as will also be described further below, the CDSP method may be enhanced with an obstacle avoidance strategy by using a gradient of a repulsive potential function. Because the semi-contracting or partially contracting trajectories being modeled are globally converging to a goal location (i.e., the location of an object being reached for), the addition of local repulsive potential need not change attractor behavior at the goal location.

[0020] Further, because the demonstrations are not directly performed by moving the robot arm, some robots may not be able to follow the parts of trajectories generated by the learned dynamics. To circumvent this problem, a low-level motion planning algorithm or inverse kinematics may be used for implementation of the trajectories generated by the learned model on a specific robot platform.

[0021] In some embodiments, CDSP is robust to abrupt changes in trajectories that may appear due to disturbances, such as sensor failures or malfunctioning of the robot **110**. The learned motion model may recreate paths that converge to the goal locations in spite of such disturbances. Another other advantage of CDSP, in some embodiments, is that the trajectories may be learned based on a single demonstration, although learning paths based on multiple demonstrations can be beneficial, especially in the case of a bad single demonstration.

[0022] FIG. **2** is a flow diagram of a method **200** of learning semi-contracting or partially contracting nonlinear dynamics from one or more demonstrations for the purpose of robot motion planning, according to one or more embodiments of this disclosure.

[0023] As shown, at block **210**, the learning system **100** may observe one or more demonstrations of a user performing a task. For example, and not by way of limitation, the task may be a reaching task, such as for loading or unloading a dishwasher, placing food in a microwave, opening a door, or picking up something. In some embodiments, this may include recording joint positions of the user, which may be done with a three-dimensional (3D) video camera such as Microsoft® Kinect® for Windows®. At block **220**, the learning system **100** may obtain training data describing the user's movements when performing the demonstrations. For example, and not by way of limitations, this training data may include estimates of position, velocity, and acceleration of the user's hand, joints, or other body part. In other words, the learning system **100** may determine positions and movements of the user based on the demonstrations. In some embodiments, the estimates may be obtained by computing them through application of a local Kalman filter to the joint positions of the user.

[0024] For encoding motions of the demonstrations, there may exist a state variable $x(t)=[p(t)^T, v(t)^T]^T \in \mathbb{R}^{2d}$, where $p(t) \in \mathbb{R}^d$ is the position and $v(t) \in \mathbb{R}^d$ is the velocity of a point in d dimensions at time t. Let a set of N demonstrations $\{D_i\}_{i=1}^N$ be a set of solutions to the dynamic model $\dot{x}(t)=f(x(t))$, where f: $\mathbb{R}^{2d} \rightarrow \mathbb{R}^d$ is a nonlinear, continuously differentiable, autonomous function. Each demonstration may correspond to a reaching motion ending at $x^*=[g^T, 0_{1\times d}]^T$, where $g \in \mathbb{R}^d$ is the goal location. Each demonstration may be associated with a set of trajectories of the states $\{x(t)\}_{t=1}^{t=T}$ and a set of trajectories of the state derivatives $\{\dot{x}(t)\}_{t=1}^{t=T}$ from time t=0 to t=T.

[0025] In the case of point-to-point reaching motions, the trajectories of a human hand can start from various initial locations and end at the goal location. Additionally, in some embodiments, the velocity and acceleration is zero at the goal location.

[0026] In some embodiments, the preprocessing unit **120** may perform blocks **210-220** of the method **200**.

[0027] At block **230**, the learning system **100** may define parameters of a NN or GMM to be used as a model. These parameters may define a structure for the NN or GMM and may include, for example, the number of neurons in the hidden layer of the NN or a number of Gaussians in the GMM. The parameters may vary based on implementation and may be a design choice. For instance, as the number of neurons in the hidden layer increases, the learning system **100** may take longer to compute trajectories for the robot **110**, but the trajectories may be more precise and may thus result in improved performance of the task by the robot **110**.

[0028] At block **240**, the learning system **100** may compute weights for initializing the NN or GMM without constraints, based at least in part on the training data.

[0029] As discussed above, the learning system **100** may utilize contraction analysis to analyze exponential stability of nonlinear systems. A non-linear, non-autonomous system may have the form $\dot{x}=f(x,t)$ (hereinafter "Formula 1"), where $x(t)\in\mathbb{R}^n$ is a state vector, and f: $\mathbb{R}\rightarrow\mathbb{R}^n$ is a continuously differentiable nonlinear function. In this case, the relation

$$\delta\dot{x} = \frac{\partial f(x,t)}{\partial x}$$

δx holds, where δx is an infinitesimal virtual displacement in fixed time. The squared virtual displacement between two trajectories of Formula 1 in a symmetric, uniformly positive definite metric $M(x,t)\in\mathbb{R}^{n\times n}$ may be given by $\delta x^T M(x,t)\delta x$, and its time derivative may be given by

$$\frac{d}{dt}(\delta x^T M(x,t)\delta x) = \delta x^T\left(\frac{\partial f^T}{\partial x}M(x,t) + \dot{M}(x,t) + M(x,t)\frac{\partial f}{\partial x}\right)\delta x.$$

[0030] If the inequality

$$\frac{\partial f^T}{\partial x}M(x,t) + \dot{M}(x,t) + M(x,t)\frac{\partial f}{\partial x} \leq 0\forall t,$$

$\forall x$ is satisfied, then the system of Formula 1 may be deemed to be semi-contracting. Further, the trajectory of Formula 1 may converge to a single trajectory regardless of initial conditions.

[0031] If the inequality

$$\frac{\partial f^T}{\partial x}M(x,t) + \dot{M}(x,t) + M(x,t)\frac{\partial f}{\partial x} \leq -2\gamma M(x,t),\ \forall t,$$

$\forall x$ is satisfied for a strictly positive constant γ, then the system of Formula 1 may be deemed to be contracting with the rate γ. Further, the trajectory of Formula 1 may converge to a single trajectory regardless of initial conditions.

[0032] For an auxiliary system of the form $\dot{y}=f(y,x,t)$ (hereinafter "Formula 2"), where $y(t)\in\mathbb{R}^n$ is an auxiliary variable, if the inequality

$$\frac{\partial f^T}{\partial y}M(y,t) + \dot{M}(y,t) + M(y,t)\frac{\partial f}{\partial y} \leq -2\gamma M(y,t),\ \forall t,$$

[0033] $\forall y$ is satisfied for a strictly positive constant γ, then the auxiliary system of Formula 2 may be deemed to be contracting and the system of Formula 1 may be deemed to be partially contracting. Further, if any particular solution of the auxiliary system of Formula 2 verifies a smooth specific property, then all the trajectories of the system of Formula 1 verify this specific property exponentially.

[0034] In some embodiments, a set of N demonstrations $\{D_i\}_{i=1}^N$ performed by the user are solutions to an underlying dynamic model governed by the first-order differential equation $\dot{x}(t)=f(x(t))$ (hereinafter "Formula 3"), where the state variable $x\in\mathbb{R}^n$, and f: $\mathbb{R}\rightarrow\mathbb{R}^n$ is a nonlinear continuous and continuously differentiable function.

[0035] Each demonstration may include trajectories of the states $\{x(t)\}_{t=1}^{t=T}$ and trajectories of the state derivatives $\{\dot{x}(t)\}_{t=1}^{t=T}$ from time t=0 to t=T. Because the state trajectories of the demonstrations of a specific stable DS may exponentially converge to a single trajectory or a single point (i.e., the goal location), the system defined in Formula 3 may be considered a globally contracting system.

[0036] In some embodiments of the learning system **100**, the nonlinear function f is modeled using a NN given by $f(x(t))=W^T\sigma(U^Ts(t)+\epsilon(s(t))$ (hereinafter "Formula 4"). In Formula 4, $s(t)=[x(t)^T, 1]^T\in\mathbb{R}^{n+1}$ is an input vector to the NN;

$$\sigma(U^Ts(t)) = \left[\frac{1}{1+\exp((-U^Ts(t))_1)}, \cdots \frac{1}{1+\exp((-U^Ts(t))_i)}, \cdots \frac{1}{1+\exp\left((-U^Ts(t))_{n_h}\right)}\right]^T$$

[0037] is a vector-sigmoid activation function, and $(U^Ts(t))_i$ is the ith element of the vector $(U^Ts(t), U\in\mathbb{R}^{(n+1)\times n_h}$; the elements of $W\in\mathbb{R}^{n_h\times n}$ are bounded constant-weight matrices; $\epsilon(s(t))\in\mathbb{R}^n$ is a function reconstruction error that goes to zero after the NN is fully trained; and n_h is the number of neurons in the hidden layer of the NN.

[0038] In some embodiments of the learning system **100**, the nonlinear function f is modeled using a GMM given by $f(x(t))=\Sigma_k h_k(x(t))(A_k x(t)+b_k)+\epsilon(x(t))$ (hereinafter "Formula 5"). In Formula 5,

$$h_k(x(t)) = \frac{p(k)p(x\mid k)}{\Sigma_k p(k)p(x\mid k)}$$

is the scalar weight associated with the kth Gaussian such that $\Sigma_k h_k(x(t))=1$ and $0\leq h_k(x(t))\leq 1$, p(k) is the prior probability, and

$$A_k = \Sigma_{k_{\dot{x}x}}(\Sigma_{k_x})^{-1},\ b_k = \mu_{k_{\dot{x}}} - A_k\mu_{k_x},\ \mu_k = \left[\mu_{k_x}^T, \mu_{k_{\dot{x}}}^T\right]^T \text{ and}$$

$$\Sigma_k = \begin{bmatrix} \Sigma_{k_x} & \Sigma_{k_{\dot{x}x}} \\ \Sigma_{k_{\dot{x}x}} & \Sigma_{k_{\dot{x}}} \end{bmatrix}$$

are the mean and covariance of the kth Gaussian, respectively.

[0039] Given the one or more demonstrations, the learning system **100** may learn the function f, which is modeled using a NN or GMM under contraction conditions. This may enable the learning system **100** to generate converging trajectories, governed by a stable DS and starting from a given arbitrary initial condition. As a result, the learning system **100** may cause the robot **110** to execute such a trajectory, by performing the task demonstrated in the one or more demonstrations, given arbitrary initial conditions.

[0040] In some embodiments, the constrained optimization problem to be solved by the learning system **100** to train the semi-contracting NN may be $\{\hat{W},\hat{U}\}=\mathrm{argmin}_{W,U}\{\alpha E_D+\beta E_W\}$ (hereinafter "Formula 6"), such that

$$\frac{\partial f^T}{\partial x}M + M\frac{\partial f}{\partial x} \le 0,$$

M>0 (hereinafter "Formula 7"). In Formula 6 and Formula 7, $E_D=\Sigma_{i=1}^{N}[y_i-a_i]^T[y_i-a_i]$ may be the sum of squared errors; $a_i\in\mathbb{R}^n$ and $y_i\in\mathbb{R}^n$ may respectively represent the end location and the NN's output corresponding to the ith demonstration; E_W may be the sum of the squares of the NN weights; α and β may be parameters of regularization; and $M\in\mathbb{R}^{n\times n}$ may represent a constant positive symmetric matrix. The learning system may compute the Jacobian

$$\frac{\partial f}{\partial x} \text{ as } \frac{\partial f}{\partial x} = W^T\frac{\partial\sigma(U^Ts)}{\partial x} = W^T[\Sigma'(U^Ts)]U_x^T.$$

In the above, for any $b\in\mathbb{R}^{n_h}$, $\Sigma'(b)\in\mathbb{R}^{n_h\times n_h}$ may be a diagonal matrix given by $\Sigma'(b)=\mathrm{diag}(\sigma(b_1)(1-\sigma(b_1)), \sigma(b_2)(1-\sigma(b_2)), \ldots, \sigma(b_p)(1-\sigma(b_{n_h})))$ (hereinafter "Formula 8"); and $U_x\in\mathbb{R}^{n\times n_h}$ may be a sub-matrix of U formed by taking the first n rows of U.

[0041] In some embodiments, the constrained optimization problem to be solved by the learning system **100** to train the partially contracting GMM may be $\{\hat{\theta}_G\}=\mathrm{argmin}_{\theta_G}\{\alpha E_D+\beta E_W\}$ (hereinafter "Formula 9"), such that

$$\frac{\partial f^T}{\partial x}M(y) + \dot{M}(y, t) + M(y)\frac{\partial f}{\partial y} \le 0,\ M(y) > 0,\ A_kx^* + b_k = 0,\ \forall\ y, k$$

(hereinafter "Formula 10"), where x* is the desired equilibrium point of the GMM. In Formula 9 and Formula 10, $E_D=\Sigma_{i=1}^{N}[y_i-a_i]^T[y_i-a_i]$ may be the sum of squared errors; $a_i\in\mathbb{R}^n$ and $y_i\in\mathbb{R}^n$ may respectively represent the end location and the GMM's output corresponding to the ith demonstration; E_W may be the sum of the squares of the GMM parameters; α and β may be parameters of regularization; and $M(y)\in\mathbb{R}^{n\times n}$ may represent a uniformly positive symmetric matrix.

[0042] Formula 7 and Formula 10 are examples of contraction constraints incorporated into the learning process of the learning system **100**. They are derived from contraction theory, which studies the behavior of trajectories. This constraint on the Jacobian, which is the first order derivative of the function f with respect to the state, may ensure that all the trajectories learned will converge to the goal location as well as achieve zero velocity at the goal location regardless of initial conditions.

[0043] At block **250**, based on the initialization of the NN or GMM in block **240**, the learning system **100** may learn the NN or GMM with contraction analysis constraints. Constraints may embody motion characteristics, or motion limitations, of the task that was demonstrated. For example, and not by way of limitation, if the task is a reaching task, the user's hand likely reached a specific velocity (e.g., a zero velocity) at the object being reached for during the demonstrations, and if the task is polishing a table, the task likely includes some periodicity as the user rubbed the table in a circular motion. In some embodiments, learning with contraction analysis constraints may be achieved by solving an optimization problem, as described below.

[0044] The optimization problem defined in Formula 6 and Formula 7 above can be rewritten as $\{\hat{W},\hat{U}\}=\mathrm{argmin}_{W,U}\{\alpha\Sigma_{i=1}^{N}[y_i-a_i]^T[y_i-a_i]^T[y_i-a_i]+\beta(\mathrm{tr}(W^TW)+\mathrm{tr}(U^TU))\}$ (hereinafter "Formula 11"), such that $U_x[\Sigma'(U^Ts)]^TWM+MW^T[\Sigma'(U^Ts)]U_x^T\ge0$, M>0 (hereinafter "Formula 12").

[0045] As shown below, the nonconvex constraints of Formula 12 can be relaxed to LMI constraints, which may be used by the learning system **100** to update the NN. It can be shown that the constraints defined in Formula 12 may be always satisfied if the following constraints are satisfied: $n=n_h$, $U_x>0$, $W<0$, $M>0$ (hereinafter "Formula 13").

[0046] The sigmoid function σ( ) is in the range [0,1], and thus the derivative of σ( )(1−σ( )) may have upper and lower bounds given by 0≤σ( )(1−σ( ))≤0.25 (hereinafter "Formula 14"). Using Formula 14 and the fact that Σ'( ) is given by Formula 8, each diagonal element of the matrix $\Sigma'(U^Ts)$ may be lower bounded by 0. The lower bound of the whole matrix may be given by $E'(U^Ts)\ge0$ (hereinafter "Formula 15"). Multiplying MW^T on the left and U_x^T on the right of Formula 15 yields $MW^T[\Sigma(U^Ts)]U_x^T\ge0$ (hereinafter "Formula 16") and $U_x[\Sigma'(U^Ts)]^TWM\le0$ (hereinafter "Formula 17").

[0047] Given Formula 16 and Formula 17, $U_x[\Sigma'(U^Ts)]^TWM$ may be upper bounded as $U_x[\Sigma'(U^Ts)]^TWM+MW^T[\Sigma'(U^Ts)]U_x^T\le0$ (hereinafter "Formula 18"). If the constraints defined in Formula 13 hold, as presumed above, then Formula 13 and Formula 18 together may yield $U_x[\Sigma'(U^Ts)]^TWM+MW^T[\Sigma'(U^Ts)]U_x^T\le0$ (hereinafter "Formula 19"). Thus, the constraint of Formula 12, being equal to Formula 19, may be satisfied where Formula 13 is true.

[0048] As shown below, the optimization problem defined in Formula 9 and Formula 10 above can be rewritten as $\{\hat{W},\hat{U}\}=\mathrm{argmin}_{W,U}\{\alpha\Sigma_{i=1}^{N}[y_i-a_i]^T[y_i-a_i]+\beta E_W\}$ (hereinafter "Formula 20"), such that $A_k^TM(y)+M_k(y)+M(x)A_k\le-\gamma M(y)$, $M(y)>0$, $A_kx^*+b_k=0$, ∀y, k (hereinafter "Formula 21"), where the ijth element of the matrix $\dot{M}_k(y)$ is given by

$$\dot{M}_{k_{ij}}(y) \triangleq \frac{dM_{ij}(y)}{dy}(A_ky+b_k).$$

[0049] Given the Jacobean

$$\frac{\partial f}{\partial y} \triangleq \Sigma_k\{h_k(x)A_k\}$$

and the decomposition of the contraction metric $\dot{M}(y)=\Sigma_kh_k(x)M_k(y)$, Formula 10 can be rewritten as $\Sigma_kh_k(x)\{A_k^TM_k(y)+\dot{M}_k(y)+M(x)A_k\}\le-\gamma M(y)$, $M(y)>0$, $A_kx^*+b_k=0$, ∀y, k (hereinafter "Formula 22"). Using the facts $\Sigma_kh_k(x(t))=1$ and $0\le h_k(x(t))\le1$, it may be shown that Formula 10 is satisfied where Formula 21 is satisfied. Note that, during implementation, in some embodiments, the constraint in Formula 21 may be evaluated at x since the GMM is partially contracting and the trajectories y(t) and x(t) converge to each other exponentially.

[0050] Note that, in some embodiments, Formula 21 depends on the state variable and must be enforced at every point in the state space, rendering the optimization problem intractable in practice. As shown below, the above noted state-dependence issue of the condition in Formula 21 may be overcome by rewriting it as $\overline{G}_k \leq 0$, $A_k x^* + b_k = 0$, $\forall k$ (hereinafter "Formula 23").

[0051] On defining the matrices $G_k \triangleq A_k^T M(y) + \dot{M}_k(y) + M(x)A_k + \gamma M(y)$, the condition in Formula 21 may be rewritten as $z^T G_k z \leq 0$, $\forall z$, where $z \in \mathbb{R}^n$ is a vector of indeterminates. By the way of sum of squares decomposition, it can be shown that $z^T G_k z = m(x,z)^T \overline{G}_k m(x,z)$, where $m(x,z) \in \mathbb{R}^{\bar{n}}$, is a vector of monomials in the elements of x and z; the elements of the matrix $\overline{G}_k$ are polynomials in the elements of the unknown parameters and may be obtained by coefficient matching.

[0052] Blocks **230-250**, described above, may be performed by the training unit **130** in some embodiments. At block **260**, the method **200** may exit, having trained the NN with contraction analysis constraints.

[0053] FIG. **3** is a flow diagram of a method **300** for causing a robot to perform the task, according to one or more embodiments of this disclosure. This method **300** may be executed by the learning system **100** after having executed the method **200** of FIG. **2**.

[0054] The trajectory generated by the semi-contracting NN or the partially contracting GMM defined above does not take obstacles into consideration. In other words, the feedback being considered by the NN or GMM of the learning system **100**, as described above, may be only the current state of the robot **110**. However, some embodiments of the learning system **100** may also execute obstacle avoidance in performing reaching tasks.

[0055] To this end, at block **310**, the learning system **100** may obtain locations of one or more obstacles o_i in a workspace of the robot **110**; an initial location of an end effector x(0) (e.g., a point on the robot's hand) intended to reach the goal location, to be treated as an origin; and the goal location x_d of the end effector. At block **320**, the learning system **100** may translate the origin to the goal location. In some embodiments, the learned NN or GMM generates trajectories to the origin. Thus, this translation may be performed to generate trajectories to the goal location instead. Blocks **310-320** may be implemented by various means known in the art.

[0056] At block **330**, the learning system **100** may implement obstacle avoidance.

[0057] In some embodiments, implementing obstacle avoidance includes computing a size of a domain of influence D_i^* for each obstacle. For instance, the learning system **100** may use an artificial repulsive potential field in the workspace of the robot **110**, in addition to the semi-contracting dynamics learned above. The repulsive potential V_r for the ith obstacle and the origin may be given by

$$V_{ri}(x) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{d_i(x)} - \frac{1}{D_i^*}\right)^2, & d_i(x) \leq D_i^* \\ 0, & d_i(x) > D_i^* \end{cases}. \quad \text{(hereinafter "Formula 24")}$$

[0058] The gradient of Formula 24 with respect to a current state x of the robot **110** may be given by

$$\nabla_x V_{ri}(x) = \begin{cases} \eta\left(\frac{1}{D_i^*} - \frac{1}{d_i(x)}\right)\frac{1}{d_i^2}\nabla_x d_i(x), & d_i(x) \leq D_i^* \\ 0, & d_i(x) > D_i^* \end{cases}. \quad \text{(hereinafter "Formula 25")}$$

In the above, $d_i = \|x - o_i\|_2$ may be the Euclidean distance from x to the location of the ith obstacle o_i; D_i^* may be the size of a domain of influence of the ith obstacle; $\eta \in \mathbb{R}^+$ may be a positive constant; and $\nabla_x d_i(x)$ may denote the derivative of $d_i(x)$ with respect to x.

[0059] In some embodiments, the negative gradient of Formula 24, given by the negative of Formula 25 results in a repulsive force acting on the robot **110**. The repulsive force may drive the robot **110** away from the obstacles and can thus be viewed as a force that acts along with an attractive force to drive the robot **110** to the goal location. The attractive force may be provided by the semi-contracting NN or the partially contracting GMM. Thus, where n_0 is the number of obstacles to be avoided, the combined dynamics $f_c(\,)$ may be described by $\dot{x} = f_c(x(t) = f(x(t)) - \Sigma_i \nabla_x V_{ri}(x(t))$, for $i = \{1, \ldots, n_0\}$ (hereinafter "Formula 26").

[0060] In some embodiments, implementing obstacle avoidance may involve use of a differential equation that models the obstacle avoidance. According to a study of human behavioral dynamics, an additional term for n_0 obstacles causes a change in acceleration that is given by f_{obs} as follows:

$$f_{obs} = (x(t)) = \gamma\sum_{i=1}^{n_0} R(t)v(t)\varphi_i(t)\exp(-\beta\varphi_i(t))$$

where γ and β are positive scalar constants;

$$\varphi_i(t) = \cos^{-1}\left(\frac{(o_i - p(t))^T v(t)}{\|o_i - p(t)\|\|v(t)\|}\right)$$

is the steering angle between $(o_i - p(t)$ and v(t); o_i is the position of the ith obstacle; R(t) is the rotation matrix that defines a ninety-degree rotation about the axis $r(t) = (o_i - p(t) \times v(t)$. Thus, the combined dynamics $f_c(\,)$ are described by

$$\dot{x}(t) = f_c(x(t)) = f(x(t)) + \begin{bmatrix} 0_{d\times 1} \\ f_{obs}(x(t)) \end{bmatrix}. \quad \text{(hereinafter "Formula 27")}$$

[0061] In some embodiments, for $M = cI_{2d\times 2d}$, where $c \in \mathbb{R}^+$ is a strictly positive scalar constant, it can be shown that all the trajectories of the combined dynamics in Formula 27 converge to the goal location x*. Based on this, all trajectories of the combined dynamics in Formula 27 may converge to the goal location. Thus, for all t, $\dot{x}(t)$ is not equal to zero anywhere in the state space except at the goal location x*. Therefore, in some embodiments, there are no local minima present in the state space and the goal location x* is the global minimum.

[0062] In some embodiments, the combined dynamics given by Formula 27 may provide the robot **110** with a combination of two forces, one moving it away from the obstacles and the other toward the goal location.

[0063] At block **340**, the learning system **100** may generate a trajectory based on the learned NN or GMM along with obstacle avoidance, as described in Formula 26 above.

[0064] It can be shown that, where T1(t) is a trajectory of the globally semi-contracting system of Formula 4 and T2(t) is a trajectory of the combined dynamics of Formula 26, the smallest distance, defined by $S(t) \triangleq \int_{T_1}^{T_2} \|\delta x\|$, between T1(t) and T2(t) satisfies

$$S(t) \leq S(t_0)e^{-\gamma_1(t-t_0)} + \frac{1-e^{-\gamma_1(t-t_0)}}{\gamma_1} sup_{x,t}\|d\| \forall\ t \geq t_0, \quad \text{(hereinafter “Formula 28”)}$$

[0065] and as

$$t \to \infty,\ S(t) \leq sup_{x,t}\frac{\|d\|}{\gamma_1},\ \text{where } d(x(t)) = -\Sigma_i \nabla_x V_{ri}(x)$$

for $i=\{1, \ldots, n_0\}$. Thus, in some embodiments, there exists an upper bound on the distance between the trajectory of the learned contracting model and the trajectory of the combined dynamics (i.e., the learned contracting model using an obstacle avoidance approach described herein). As a result, addition of the obstacle avoidance need not lead to the trajectory to monotonically diverge away from the trajectory of the original learned contracting model.

[0066] A proof of this involves the following. Differentiating the distance S(t) yields the differential inequality $\dot{S} \leq -\gamma S + \|d\|$, whose solution is given by S(t). Let $\overline{S}(t)$ be the solution to the differential equation $\dot{\overline{S}} = -\gamma\overline{S} + \|d\|$. The trajectory $\overline{S}(t)$ is given by $\overline{S}(t) = \overline{S}(t_0)e^{-\gamma_1(t-t_0)} + \int_{t_0}^{t} e^{-\gamma_1(t-\tau)}\|d(x(t))\|d\tau$. Based on Khalil's comparison lemma, $S(t) \leq \overline{S}(t)$, which implies $S(t) \leq \overline{S}(t_0)e^{-\gamma_1(t-t_0)} + \int_{t_0}^{t} e^{-\gamma_1(t-\tau)}\|d(x(t)\|d\tau$ (hereinafter “Formula 29”). Taking the supremum of $\|d(x(t))\|$ out of the integral in Formula 29 results in Formula 28. As $t \to \infty$, the exponential terms decay and the bound is given by

$$S(t) \leq sup_{x,t}\frac{\|d\|}{\gamma_1}.$$

[0067] Blocks **330-340** above may be performed by the trajectory generation unit **140** of the learning system **100**. At block **350**, the learning system **100** may convert the trajectory generated above from the Cartesian space into a trajectory in the joint space of the robot **110**. In some embodiments, this may be executed through the use of IKFast, a robot kinematics solver, or by some other solver. At block **360**, the learning system **100** may implement the trajectory, such as by using a low-level joint controller to control the robot **110** according to the trajectory in joint space. Black **350-360** may be performed by the motion planning unit **150** of the learning system **100**. At block **370**, the method **300** may exit, with the robot **110** having performed the task through moving according to the joint-space trajectory.

[0068] It will be understood by one skilled in the art that various implementations of the learning system **100** and the robot **110** may be used. For example, in one embodiment, the demonstrations may each include a human subject reaching for a target location to pick up an object, and data describing these demonstrations may be collected using a Microsoft® Kinect® for Windows®. The learning system **100** may be implemented on a desktop computer running an Intel® i3 processor and having 8 GBs of memory. The methods described above may be coded on the desktop computer using Matlab 2014a. The learning system **100** on the desktop computer may be used to control a robot **110**, such as a Baxter robot, whose hand position in 3D Cartesian space is considered to be the state. Velocity estimates of the hand may be estimated from position measurements of the hand using a Kalman filter. There may be six neurons in the hidden layer of the NN, and the NN weights or the GMM parameters of the constrained optimization algorithm may be initialized to weights or parameters obtained by learning the NN or GMM without the constraints. In the case of the NN, the identity matrix may be used as the metric M. In the case of the GMM, the parameters of the state-dependent contraction metric may be learned from the one or more demonstrations. Matlab's fmincon function may be used to solve the optimization problem. The implementation on the robot may be achieved through IKFast to convert the resulting trajectory into a trajectory in the joint space of the robot.

[0069] Technical effects and benefits of some embodiments include the ability to learn a semi-contracting dynamic motion model in a state space is presented. The learned model may be used to generate motion trajectories of a robot based on human demonstrations. Through a CDSP, some embodiments of the learning system **100** may combine the advantages of global stability with a NN model or a GMM. In some embodiments, obstacle avoidance and motion planning may be incorporated. The global semi-contracting nature of the dynamics may make the goal location globally attractive, thus causing the dynamics to be robust to perturbations and sensor faults. Further, some embodiments of the learning system **100** may be platform-agnostic and thus compatible with various types of robots **110**, including robots **110** from various manufacturers.

[0070] FIG. **4** illustrates a block diagram of a computer system **400** for use in implementing a learning system **100** or method according to some embodiments. The learning systems **100** and methods described herein may be implemented in hardware, software (e.g., firmware), or a combination thereof. In some embodiments, the methods described may be implemented, at least in part, in hardware and may be part of the microprocessor of a special or general-purpose computer system **400**, such as a personal computer, workstation, minicomputer, or mainframe computer. For instance, as described above, the learning system **100** may be implemented on a desktop computer in communication with the robot **110**.

[0071] In some embodiments, as shown in FIG. **4**, the computer system **400** includes a processor **405**, memory **410** coupled to a memory controller **415**, and one or more input devices **445** and/or output devices **440**, such as peripherals, that are communicatively coupled via a local I/O controller **435**. These devices **440** and **445** may include, for example, a printer, a scanner, a microphone, and the like. Input devices such as a conventional keyboard **450** and mouse **455** may be coupled to the I/O controller **435**. The I/O controller **435** may be, for example, one or more buses or other wired or wireless connections, as are known in the art. The I/O controller **435** may have additional elements, which are omitted for simplicity, such as controllers, buffers (caches), drivers, repeaters, and receivers, to enable communications.

[0072] The I/O devices **440**, **445** may further include devices that communicate both inputs and outputs, for instance disk and tape storage, a network interface card (NIC) or modulator/demodulator (for accessing other files, devices, systems, or a network), a radio frequency (RF) or other transceiver, a telephonic interface, a bridge, a router, and the like.

[0073] The processor **405** is a hardware device for executing hardware instructions or software, particularly those stored in memory **410**. The processor **405** may be a custom made or commercially available processor, a central processing unit (CPU), an auxiliary processor among several processors associated with the computer system **400**, a semiconductor based microprocessor (in the form of a microchip or chip set), a microprocessor, or other device for executing instructions. The processor **405** includes a cache **470**, which may include, but is not limited to, an instruction cache to speed up executable instruction fetch, a data cache to speed up data fetch and store, and a translation lookaside buffer (TLB) used to speed up virtual-to-physical address translation for both executable instructions and data. The cache **470** may be organized as a hierarchy of more cache levels (L**1**, L**2**, etc.).

[0074] The memory **410** may include one or combinations of volatile memory elements (e.g., random access memory, RAM, such as DRAM, SRAM, SDRAM, etc.) and nonvolatile memory elements (e.g., ROM, erasable programmable read only memory (EPROM), electronically erasable programmable read only memory (EEPROM), programmable read only memory (PROM), tape, compact disc read only memory (CD-ROM), disk, diskette, cartridge, cassette or the like, etc.). Moreover, the memory **410** may incorporate electronic, magnetic, optical, or other types of storage media. Note that the memory **410** may have a distributed architecture, where various components are situated remote from one another but may be accessed by the processor **405**.

[0075] The instructions in memory **410** may include one or more separate programs, each of which comprises an ordered listing of executable instructions for implementing logical functions. In the example of FIG. **4**, the instructions in the memory **410** include a suitable operating system (OS) **411**. The operating system **411** essentially may control the execution of other computer programs and provides scheduling, input-output control, file and data management, memory management, and communication control and related services.

[0076] Additional data, including, for example, instructions for the processor **405** or other retrievable information, may be stored in storage **420**, which may be a storage device such as a hard disk drive or solid state drive. The stored instructions in memory **410** or in storage **420** may include those enabling the processor to execute one or more aspects of the learning systems **100** and methods of this disclosure.

[0077] The computer system **400** may further include a display controller **425** coupled to a display **430**. In some embodiments, the computer system **400** may further include a network interface **460** for coupling to a network **465**. The network **465** may be an IP-based network for communication between the computer system **400** and an external server, client and the like via a broadband connection. The network **465** transmits and receives data between the computer system **400** and external systems. In some embodiments, the network **465** may be a managed IP network administered by a service provider. The network **465** may be implemented in a wireless fashion, e.g., using wireless protocols and technologies, such as WiFi, WiMax, etc. The network **465** may also be a packet-switched network such as a local area network, wide area network, metropolitan area network, the Internet, or other similar type of network environment. The network **465** may be a fixed wireless network, a wireless local area network (LAN), a wireless wide area network (WAN) a personal area network (PAN), a virtual private network (VPN), intranet or other suitable network system and may include equipment for receiving and transmitting signals.

[0078] Learning systems **100** and methods according to this disclosure may be embodied, in whole or in part, in computer program products or in computer systems **400**, such as that illustrated in FIG. **4**.

What is claimed is:

1. A computer-implemented method comprising:

recording one or more demonstrations of a task performed by a user;

determining, based on the one or more demonstrations, movements of one or more joints of the user;

learning, by a computer processor, based on the movements of the one or more joints of the user, a learned model incorporating one or more contraction analysis constraints, the one or more contraction analysis constraints representing motion characteristics of the task, wherein the learned model is at least one of a neural network and a Gaussian mixture model;

determining a first initial position of a robot; and

determining a first trajectory of the robot to perform the task, the determining based at least in part on the learned model and the first initial position.

2. The computer-implemented method of claim **1**, further comprising:

determining a second initial position of the robot for performing the task, wherein the first initial position differs from the second initial position; and

determining a second trajectory of the robot to perform the task, based at least in part on the learned model and the second initial position;

wherein the first trajectory of the robot and the second trajectory of the robot converge to a common goal location.

3. The computer-implemented method of claim **1**, wherein the learned model is the neural network, and wherein learning the learned model incorporating the one or more contraction analysis constraints comprises:

initializing the neural network by solving an optimization problem to generate one or more weights of the neural network;

generating an updated optimization problem based on the optimization problem and the one or more contraction analysis constraints; and

updating the neural network by solving the updated optimization problem.

4. The computer-implemented method of claim **1**, wherein the first trajectory is in a joint space of the robot.

5. The computer-implemented method of claim **1**, further comprising:
defining a size of a domain of influence for an object in a workspace of the robot; and
generating a second trajectory for performing the task, based at least in part on the learned model, the first initial position, and the size of the domain of influence of the object;
wherein the second trajectory avoids the object.

6. The computer-implemented method of claim **5**, wherein the first trajectory and the second trajectory converge to a common goal location.

7. The computer-implemented method of claim **1**, wherein the one or more contraction analysis constraints require a first velocity at a goal location of the task.

8. A system comprising:
a memory having computer readable instructions; and
one or more processors for executing the computer readable instructions, the computer readable instructions comprising:
recording one or more demonstrations of a task performed by a user;
determining, based on the one or more demonstrations, movements of one or more joints of the user;
learning, based on the movements of the one or more joints of the user, a learned model incorporating one or more contraction analysis constraints, the one or more contraction analysis constraints representing motion characteristics of the task, wherein the learned model is at least one of a neural network and a Gaussian mixture model;
determining a first initial position of a robot; and
determining a first trajectory of the robot to perform the task, the determining based at least in part on the learned model and the first initial position.

9. The system of claim **8**, the computer readable instructions further comprising:
determining a second initial position of the robot for performing the task, wherein the first initial position differs from the second initial position; and
determining a second trajectory of the robot to perform the task, based at least in part on the learned model and the second initial position;
wherein the first trajectory of the robot and the second trajectory of the robot converge to a common goal location.

10. The system of claim **8**, wherein the learned model is the neural network, and wherein learning the learned model incorporating the one or more contraction analysis constraints comprises:
initializing the neural network by solving an optimization problem to generate one or more weights of the neural network;
generating an updated optimization problem based on the optimization problem and the one or more contraction analysis constraints; and
updating the neural network by solving the updated optimization problem.

11. The system of claim **8**, wherein the first trajectory is in a joint space of the robot.

12. The system of claim **8**, the computer readable instructions further comprising:
defining a size of a domain of influence for an object in a workspace of the robot; and
generating a second trajectory for performing the task, based at least in part on the learned model, the first initial position, and the size of the domain of influence of the object;
wherein the second trajectory avoids the object.

13. The system of claim **12**, wherein the first trajectory and the second trajectory converge to a common goal location.

14. The system of claim **8**, wherein the one or more contraction analysis constraints require a first velocity at a goal location of the task.

15. A computer program product for transferring a skill to a robot, the computer program product comprising a computer readable storage medium having program instructions embodied therewith, the program instructions executable by a processor to cause the processor to perform a method comprising:
recording one or more demonstrations of a task performed by a user;
determining, based on the one or more demonstrations, movements of one or more joints of the user;
learning, based on the movements of the one or more joints of the user, a learned model incorporating one or more contraction analysis constraints, the one or more contraction analysis constraints representing motion characteristics of the task, wherein the learned model is at least one of a neural network and a Gaussian mixture model;
determining a first initial position of a robot; and
determining a first trajectory of the robot to perform the task, the determining based at least in part on the learned model and the first initial position.

16. The computer program product of claim **15**, the method further comprising:
determining a second initial position of the robot for performing the task, wherein the first initial position differs from the second initial position; and
determining a second trajectory of the robot to perform the task, based at least in part on the learned model and the second initial position;
wherein the first trajectory of the robot and the second trajectory of the robot converge to a common goal location.

17. The computer program product of claim **15**, wherein the learned model is the neural network, and wherein learning the neural network incorporating the one or more contraction analysis constraints comprises:
initializing the neural network by solving an optimization problem to generate one or more weights of the neural network;
generating an updated optimization problem based on the optimization problem and the one or more contraction analysis constraints; and
updating the neural network by solving the updated optimization problem.

18. The computer program product of claim **15**, wherein the first trajectory is in a joint space of the robot.

19. The computer program product of claim **15**, the method further comprising:
defining a size of a domain of influence for an object in a workspace of the robot; and

generating a second trajectory for performing the task, based at least in part on the learned model, the first initial position, and the size of the domain of influence of the object;

wherein the second trajectory avoids the object.

20. The computer program product of claim **19**, wherein the first trajectory and the second trajectory converge to a common goal location.

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